Homework 0

CS 70, Summer 2024

Due by Friday, June 21$^{\text{st}}$ at 11:59 PM

Instructions. Start each problem on a separate page. The subparts of each problem can be on the same page. Every answer should contain a calculation or reasoning. Your answers should be clear, organized, and legible—your final submission should not include scratch work or failed attempts. You must always commit to a final answer; if multiple answers are provided, the most incorrect one will be graded.

If you are completing the homework using LaTeX, you may use the templates. Homeworks must be submitted through Gradescope. See the end of the homework for submission instructions.

Sundry. Before you start writing your final homework submission, state briefly how you worked on it (e.g., if you went to office hours, how frequently you worked on it, etc.). If you worked on the assignment in a group with other students, list their names and email addresses.

1   Administrivia

(a) Make sure you are on the course Ed and Gradescope. Find and familiarize yourself with the course website. What is its homepage's URL?

(b) Read the policies page on the course website.

    (i) Suppose a student gets the following scores on the different components of the course. Compute this student's final percent grade in the class.

- Homework 0: 100%
- Homework 1: 80%
- Homework 2: 84%
- Homework 3: 90%
- Homework 4: 77%
- Homework 5: 87%
- Homework 6: 68%
- Homework 7: 92%
- Vitamin 1: 100%
- Vitamin 2: 100%
- Vitamin 3: 95%
- Vitamin 4: 100%
- Vitamin 5: 80%
- Vitamin 6: 93%
- Vitamin 7: 100%
- Midterm: 67%
- Final: 73%

    (ii) When are homeworks released and when are they due?

    (iii) How many "drops" do you get for vitamins? How many do you get for homeworks?

    (iv) What are the date and time of the midterm exam?

    (v) What are the (tentative) date and time of the final exam?

2   Collaboration

Go to the course website and read the course policies carefully. Leave a followup on Ed if you have any questions. Are the following situations violations of course policy? Provide a short explanation for each of your answers.

(a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.

(b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.

(c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it, and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.

(d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.

(e) Heidi has completed her homework using LaTeX. Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution. Irene reads it, and after she has understood it, writes her own solution with a citation to Heidi.

(f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He cites the solutions as part of his submission.

3 Forum

Our course forum Ed is incredibly useful for Q&A and communication. We will use Ed for all important announcements. You should check it frequently.

You should also use Ed to ask engage with the staff in discussions about the course material and to discuss material with your fellow students.

(a) Read the Ed Etiquette section linked in the course policies. Explain what is wrong with the following hypothetical student question: "Can someone explain the proof of Theorem XYZ to me?" Assume Theorem XYZ is a complicated concept covered in lecture.

(b) If you have a question or concern which is not directly related to the course content, where should it be directed?

(c) Suppose you have a question about the course policies. Find the thread on Ed where you would post such a question and provide its thread number.

(d) Suppose you have a question about something covered in Lecture 1. Find the thread on Ed where you would post such a question and provide its thread number.

(e) Suppose you have a question about Question 2 from Discussion 1A. Find the thread where you would post such a question and provide its thread number.

4 Academic Integrity

Write in print or type out the following pledge and sign it.

> I pledge to uphold the university's honor code: to act with honesty, integrity, and respect for others, including their work. By signing, I ensure that all written homework I submit will be in my own words, that I will acknowledge any collaboration or help received, and that I will neither give nor receive help on any examinations.

5 Sets

(a) Consider the set $\{0.70, 1.70, 2.70, 3.70, 4.70, \ldots\}$. Write out the construction of this set in set-builder notation.

(b) For each positive natural number $k \in \mathbb{N}^+$, let $A_k = \{n \in \mathbb{N} : k \text{ divides } n\}$.

   (i) Explain what the set A_2 is.

   (ii) Explain what the set $\mathbb{N} \setminus A_2$ is.

   (iii) Consider the set

$$B = A_2 \cup A_3 \cup \ldots = \bigcup_{k=2}^{\infty} A_k.$$

   Explain what the set $\mathbb{N} \setminus B$ is.

   (iv) Provide an equivalent construction of the set $\mathbb{N} \setminus B$ using only the sets $\mathbb{N}, A_1, A_2, \ldots$ and the set difference and set intersection operations.

(c) Either prove or disprove that for any sets A and B, $|A \setminus B| = |A| - |B|$.

6 Sums and Products

(a) Find the numerical value of the expression

$$\sum_{i=1}^{20} 5 + \prod_{i=1}^{3} 2.$$

(b) Let $c_1, \ldots, c_5$ and $d_1, \ldots, d_5$ be sequences of real numbers such that

$$\sum_{i=1}^{5} c_i = 10 \qquad \text{and} \qquad \sum_{j=1}^{5} d_j = 5.$$

Find the numerical values of the following expressions.

(i) $\displaystyle\sum_{i=1}^{5} \left(2c_i + 3\right)$

(ii) $\displaystyle\sum_{i=1}^{5} 2c_i + 3$

(iii) $\displaystyle\sum_{i=1}^{5} \left(2c_i - d_i + 3\right)$

(c) For each $i, j \in \{1, \ldots, n\}$, let $x_{ij} \in \mathbb{R}$ be a real number. Find indexing values a, b, c, d such that

$$\sum_{i=1}^{n} \sum_{j=i}^{n} x_{ij} = \sum_{j=a}^{b} \sum_{i=c}^{d} x_{ij}.$$

Your indexing values can be in terms of i, j, n, etc.

(d) A closed-form expression is an expression formed by combining constants and variables with a finite number of arithmetic operations, roots, exponentials, logarithms, and trigonometric functions. Find closed-form expressions for each of the following infinite sums.

(i) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k!}$

(ii) $\displaystyle\sum_{k=2}^{\infty} \frac{3^k}{k!}$

(iii) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^{3n}}$

(iv) $\displaystyle\sum_{j \in \mathbb{N}} \frac{4^j}{(j+1)!}$

(v) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{1}{4}\right)^k$

Submission. Homeworks must be submitted through Gradescope. If you are completing your homeworks on paper, please scan the pages of your homework into a PDF using any scanner or phone application such as CamScanner. **It is your responsibility to ensure that all the work on the scanned pages is legible.**

Once you upload your submission to the Gradescope assignment, you will be prompted to select pages. **It is your responsibility to correctly select the pages of your homework corresponding to each question.** If you are having difficulties scanning, uploading, or submitting your homework, post a follow-up on the main thread corresponding to this homework on Ed.