Discussion 7D

1 Household Income

The annual household incomes of a city have an average of $60,000 and a standard deviation of $40,000.

A government organization samples 400 households at random with replacement from the city. You may assume the sample size is large enough for the central limit theorem to apply.

(a) Determine whether the histogram of annual household incomes in the city is approximately normal.

(b) Determine whether the histogram of the annual household incomes in the sample is approximately normal.

(c) Find or approximate the distribution of the average annual household income in the sample.

(d) Find or approximate the chance that the average annual household income in the sample is within $1,000 of the true average income in the city.

2   Competing Normals

Jingjia and Minsoo are attending a meeting. Let X and Y be Jingjia's and Minsoo's arrival times with respect to the start of the meeting. For example, if $X = -1.2$ and $Y = 3.4$, then Jingjia arrives to the meeting 1.2 minutes early and Minsoo arrives to the meeting 3.4 minutes late.

Suppose that X and Y are independent, $X \sim \text{Normal}(0, 2)$, and $Y \sim \text{Normal}(2, 1)$.

(a) Find the chance that both Jingjia and Minsoo are late to the meeting.

(b) Find the chance that Minsoo arrives to the meeting before Jingjia.

(c) Find the chance that Minsoo and Jingjia arrive within 1 minute of each other.

3 The Rayleigh Distribution

We say R has the Rayleigh distribution if

$$f_R(r) = re^{-\frac{1}{2}r^2}, \qquad r > 0.$$

We have seen that R is the distribution of $\sqrt{Z_1^2 + Z_2^2}$ for Z_1 and Z_2 independent standard normal random variables.

(a) Find $E[R]$ without evaluating any integrals.

(b) Find $\mathrm{Var}[R]$ without evaluating any integrals.

(c) Two shots are fired at a target and strike it at coordinates (X_1, Y_1) and (X_2, Y_2). Suppose that X_1, X_2, Y_1, Y_2 are all independent and normally distributed with identical means and unit variances.

Let D be the distance between the points where the shots hit the targets. Find $\mathrm{E}[D]$ and $\mathrm{Var}[D]$.