Discussion 5D

CS 70, Summer 2024

1 Rolling Dice

(a) Suppose we are rolling a fair 6-sided die. What is the expected number of times we have to roll before we roll a 6?

(b) Suppose we have two independent, fair n-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll?ß

2 Shuttles and Taxis at Airport

In front of terminal 3 at San Francisco Airport is a pickup area where shuttles and taxis arrive according to a Poisson distribution. The shuttles arrive at a rate $\lambda_1 = 1/20$ (i.e. 1 shuttle per 20 minutes) and the taxis arrive at a rate $\lambda_2 = 1/10$ (i.e. 1 taxi per 10 minutes) starting at 00:00. The shuttles and the taxis arrive independently.

(a) What is the distribution of the following:

(i) The number of taxis that arrive between times 00:00 and 00:20?

(ii) The number of shuttles that arrive between times 00:00 and 00:20?

(iii) The total number of pickup vehicles that arrive between times 00:00 and 00:20?

(b) What is the probability that exactly 1 shuttle and 3 taxis arrive between times 00:00 and 00:20?

(c) Given that exactly 1 pickup vehicle arrived between times 00:00 and 00:20, what is the conditional probability that this vehicle was a taxi?

(d) Suppose you reach the pickup area at 00:20. You learn that you missed 3 taxis and 1 shuttle in those 20 minutes. What is the probability that you need to wait for more than 10 mins until either a shuttle or a taxi arrives?

3 Soccer Practice

Messi and Ronaldo are practicing their penalty kicks. In each *round* of their training exercise, each player takes a penalty kick and either scores or misses. Assume (somewhat unrealistically) that each penalty is independent of every other penalty kick, with Messi missing each penalty with probability p_1 and Ronaldo missing each penalty with probability p_2. Find the following quantities:

- the expected number of rounds until the first miss of *either* player;
- the expected number of rounds until both players have missed at least one shot.

4 Cookie Jars

You have two jars of cookies, each of which starts with n cookies initially. Every day, when you come home, you pick one of the two jars randomly (each jar is chosen with probability 1/2) and eat one cookie from that jar. One day, you come home and reach inside one of the jars of cookies, but you find that is empty! Let X be the random variable representing the number of remaining cookies in non-empty jar at that time. What is the distribution of X?